Parallel Momentum and Ion Heat Transport in Stochastic Magnetic Fields

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KITP – Plasma Applications

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Outline

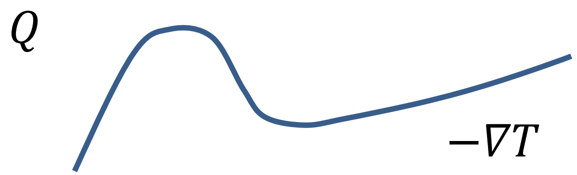
- Why ?
- Background: FGC ‘92 (Conventional Wisdom)

  Ding et. al. ‘13
- ‘Dual’ Problem: Stochastic Field Transport in Turbulence
- The Crank
- The Physics and its Implications
- Revisiting an Assumption

Danger !

Plasma Physics Ahead !!

An Observation



- BLY (1998) (layering model proto type)

BLY = '$k-\epsilon$' model + l_{mix}

- l_0 $\quad$ $l_{OZ} \sim \epsilon^{1/2} / N^{3/2}$
- l_{OZ} $\quad$ $N^2 \sim \partial_z \rho$ (emergent)

- Plasma Transport Barriers (BDT '90, Hinton '91 et. seq $\rightarrow \infty$)

'$k-\epsilon$' type models + τ_c

- $\tau_{c,0}$ $\rightarrow$ base state correlation time
- $\tau_{E\times B}$ $\rightarrow$ shearing time

$1/\tau_{E\times B} \sim V_E'$ (emergent from profiles) $\qquad 0 = \frac{q}{m}\vec{E} - \frac{\nabla P_i}{nm} + \frac{q}{mc}\vec{V} \times \langle \vec{B} \rangle$

→ Similar thinking …

Why $\langle \widetilde{B}^2 \rangle$ meets Heat, Momentum Transport?

- Samantha: Stochastic Fields $\xrightarrow{\text{dephase}}$ Shearing / Reynolds Stress / Inhibit jet production

→ Need $k_\perp^2 V_A D_M > 1/\tau_{c,0} \sim \omega_*$ to quench $\langle \tilde{V}_r \nabla_\perp^2 \tilde{\phi} \rangle$

➔ $P_{crit}(\langle b^2 \rangle)$ to form barrier

→ Focus on $\langle V_\perp \rangle$

- But $$\langle E_r \rangle = \frac{\nabla P_i}{nq} - \frac{1}{c} \langle \vec{V} \rangle \times \langle \vec{B} \rangle$$

$\langle V_E \rangle'$ heat $\langle V_\parallel \rangle$

So what of ion heat and parallel momentum ?

Why ? - Cont'd

Relevance:

Transitions → contribution to $\langle V_E \rangle'$

Intrinsic Rotation ➔ increased spontaneous rotation in H-mode

↔ heat engine

and

Stochastic fields probe resilience of barrier and staircase systems

Conventional Wisdom

- Finn, Guzdar, Chernikov ‘92 (FGC)
  - n_i , $V_\parallel$ evolution in stochastic fields (motivated by rotation damping due EML)
  - Mean field eqns:

$$\partial_t \langle V_\parallel \rangle + \partial_r \langle \tilde{V}_r \tilde{V}_\parallel \rangle = -\frac{1}{\rho} \partial_x \langle \tilde{b}_r \tilde{P} \rangle$$

$$\partial_t \langle P \rangle + \partial_r \langle \tilde{V}_r \tilde{P} \rangle = -\rho\, c_S^2\, \partial_r \langle \tilde{b}_r \tilde{V}_\parallel \rangle$$

  - QL for ‘acoustic wave response’

➔ viscous relaxation time $\tau_l \sim [c_s\, D_M \,/\, l^2]^{-1}$

$D_M = \sum_k |b_k|^2 \pi\, \delta(k_\parallel)$, ala’ RSTZ ‘66

Conventional Wisdom, Cont'd

- Nits
  - Why bother with acoustic wave?

    $\vec{B} \cdot \nabla V_{\parallel} = 0, \quad \vec{B} \cdot \nabla P = 0$ + linear response suffice

  - Structure of fluxes ??

$$\langle \tilde{b}_r \tilde{P} \rangle = -D_M \frac{\partial}{\partial r} \langle P \rangle \ ,$$

→ Residual,

non-diffusive stress for $\partial_t \langle V_{\parallel} \rangle$

$$\langle \tilde{b}_r \tilde{V}_{\parallel} \rangle = -D_M \frac{\partial}{\partial r} \langle V_{\parallel} \rangle$$

→ Convection (pinch)

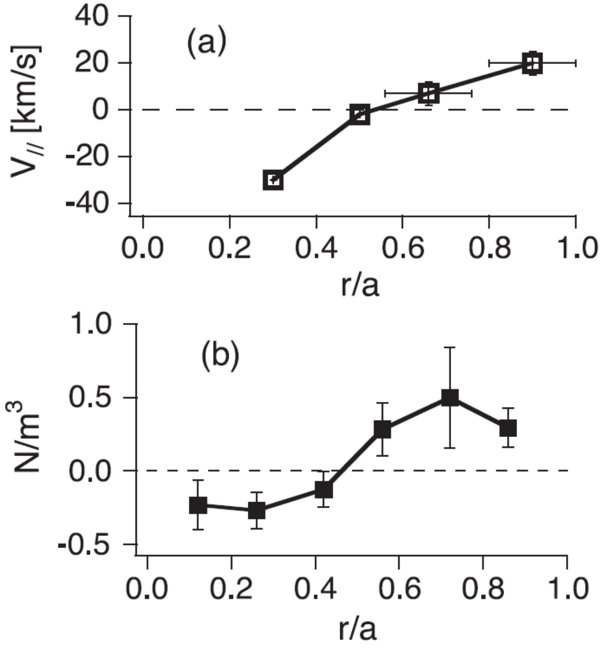
for: $\partial_t \langle P \rangle$

More Conventional Wisdom

$$\partial_t \langle V_\parallel \rangle + \partial_r \langle \tilde{V}_r \tilde{V}_\parallel \rangle = -\frac{c_s^2}{\rho}\, \partial_x \langle b_r P \rangle$$

"kinetic stress"

- W.X. Ding, et. al. PRL '13
  - Linked plasma flows in RFP to kinetic stress, via <u>direct measurement</u>
  - Mean flow profile tracks $\nabla \cdot$ (kinetic stress)



RFP = Reversed Field Pinch

Issue

- Kinetic stress of interest to transition and barrier state

- How calculate?

In QL, after FGC seek $\delta P \sim \tilde{b}\,\frac{\delta P}{\delta b} \Rightarrow \langle \tilde{b}\delta P \rangle \sim \langle \tilde{b}^2 \rangle$

<u>But what is in $\delta P \,/\, \delta b$?</u>

- Observe,

Before (Samantha): Calculate Reynolds stress via vorticity response in presence of a stochastic field

<u>Now</u>: $\boxed{\text{Calculate kinetic stress } \langle \tilde{b}\delta P \rangle \text{ via } \delta P \text{ \underline{in presence of turbulence}}}$

Issue, Cont'd

- Two problems ↔ 'dual'
  - Reynolds stress in $\langle \tilde{b}^2 \rangle$ background
  - Kinetic stress in $\langle \tilde{V}^2 \rangle$ background
- Points to significant departure from FGC and quasilinear theory !
- In spirit of Resonance Broadening, but juicier…
- Implicit: Statistics of $\tilde{b}$ → RMP induced

  $\tilde{V}$ → drift waves

  independent

The Crank

- Start from $\partial_t V_\parallel$, $\partial_t P$ equations
- Seek $\langle \tilde{b}_r \tilde{P} \rangle$, $\langle \tilde{b}_r \tilde{V}_\parallel \rangle$
- Follow quasilinear approach, BUT
- Posit an ambient ensemble of drift waves, so $\langle \tilde{V}_\perp^2 \rangle$ specified

  Assume $\langle \tilde{V}_\perp^2 \rangle$, $\langle \tilde{b}_r^2 \rangle$ quasi-Gaussian <u>and</u> statistically independent
- Calculate responses $\delta P = (\delta P/\delta b_r)\tilde{b}_r$ and $\delta V_\parallel = (\delta V_\parallel/\delta b_r)\tilde{b}_r$ (to close fluxes) by integration over <u>perturbed trajectories</u>, ala' Dupree

The Answer

(kinetic stress) $$\langle \tilde{b}_r\, \delta P \rangle = -\sum_k \left|b_{r,k}\right|^2 \underline{\frac{1}{(k_\perp^2 D_T)^2 + k_\parallel^2 c_s^2}} \left\{\rho c_s^2 k_\perp^2 D_T \frac{\partial}{\partial r}\langle V_\parallel \rangle - i k_\parallel c_s^2 \frac{\partial}{\partial r}\langle P \rangle\right\}$$

(convection) $$\langle \tilde{b}_r \delta V_\parallel \rangle = -\sum_k \left|b_{r,k}\right|^2 \underline{\frac{1}{(k_\perp^2 D_T)^2 + k_\parallel^2 c_s^2}} \left\{c_s^2 k_\perp^2 D_T \frac{\partial}{\partial r}\langle P \rangle - i k_\parallel c_s b_{r,k} c_s \frac{\partial}{\partial r}\langle V_\parallel \rangle\right\}$$

$$D_T \equiv \int \langle \tilde{V}_r \tilde{V}_r \rangle dt$$

$$\boxed{D \equiv 1/\tau_\perp^2 + k_\parallel^2 c_s^2}$$

The Physics

- Limits

$k_\parallel c_s > k_\perp^2 D_T$ → weak e.s. turbulence -- narrow regime validity

n.b. role of anisotropy !

$$\langle \tilde{b}_r \delta P \rangle \approx -D_M\, \partial \langle P \rangle / \partial r, \;\; \langle \tilde{b}_r \delta V_\parallel \rangle \approx -D_M \partial \langle V_\parallel \rangle / \partial r$$

Recovers FGC, but relevance?

- $k_\perp^2 D_T > k_\parallel c_s$ → strong electrostatic turbulence (as for pre-transition)

$$\langle \tilde{b}_r \delta P \rangle \approx -D_{st}\, \partial \langle V_\parallel \rangle / \partial r \quad , \quad \langle \tilde{b}_r \delta V_\parallel \rangle \approx -D_{st} \partial \langle P \rangle / \partial r$$

→ Viscosity → Thermal diffusivity

$$\boxed{D_{ST} = \sum_k c_S^2 \left| b_{r,k} \right|^2 / \, k_\perp^2 D_T}$$

The Physics, Cont'd

- $D_{ST} = \sum_k c_s^2 \left|b_{r,k}\right|^2 / \, k_\perp^2 D_T$ magnetic scattering with τ_{cor} set by electrostatics
- Hybrid — Magnetic / Electrostatic
- Yes, resonance broadening result, but No → not "same old, same old"

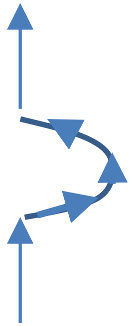
→ Structure of correlator changes from residual stress to turbulent viscosity

- FGC irrelevant to any state with finite ambient electrostatic turbulence
- FGC 'correct' to link decay of rotation to diffusion, but with incorrect diffusion coeff.

The Physics, Cont'd

- How ?

$b_{r,k}\,\partial_r \langle P \rangle$ → pressure perturbation. Balance?



→ if $k_\parallel c_S > k_\perp^2 D_T$, $\nabla_\parallel \delta P$

➔ FGC → residual stress balance perturbation

→ if $k_\parallel c_S < k_\perp^2 D_T$, $k_\perp^2 D_T \delta V_\parallel$

➔ turbulent viscosity balance perturbation

- Easily extended to sheared field geometry

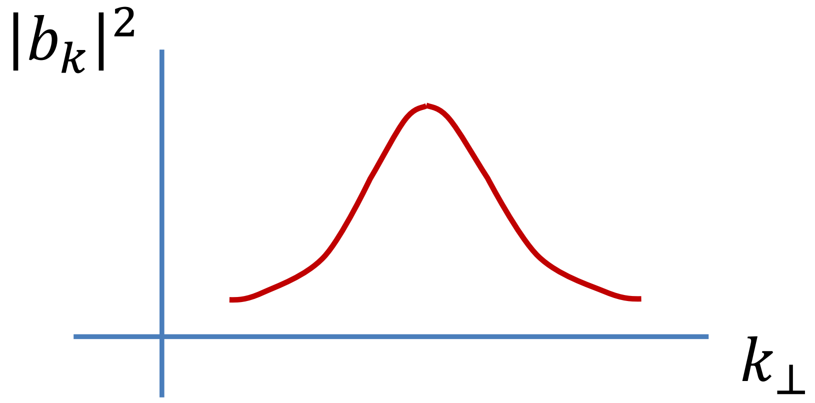
key: W_k vs $X_S = 1\,/\,c_s \tau_{ck} k_\parallel'$ (analogous to X_i)

spectral width (space) acoustic point

$W_k > X_s$ → weak
$W_k < X_s$ → strong

The Physics, Cont'd

- Infrared behavior ? $D_{ST} = \sum_k c_s^2 \left|b_{r,k}\right|^2 / k_\perp^2 D_T$
- Not resolved by magnetic shear:
- $\left|b_{r,k}\right|^2$ likely to have low-k cut-off



- Merits further study…
- May force zonal flow formation → resonance (D)

Implications / Conclusions

- Pure ‘stochastic field’ models of limited utility when turbulence present

  Need <u>both</u> of dual analyses

  → X in presence of stochastic field

  <u>and</u>

  → stochastic field in presence of turbulence

- In <u>practice</u> → <u>kinetic stress</u> is <u>turbulent viscous stress</u>

  ➔ significant effect on $\langle V_{\parallel} \rangle$

Implications / Conclusions

- Stochastic field ion heat transport also significant

- For Alfvenic case:

$$k_{\|} \underline{V_A} \quad \sim \quad k_{\perp}^2 D_T$$

so w. t. regime of greater relevance

Open Issues

- Infrared behavior

- Correlations? (with Mingyun Cao)

  – are $\tilde{b}$, turbulence uncorrelated ?

  – No → interaction develops $\langle \tilde{b}\tilde{\phi} \rangle \neq 0$

  – ala' Kadomtsev – Pogutse, impose $\nabla \cdot \vec{J} = 0$ to all orders

  – novel small scale convection cell, related to $\tilde{b}$ structure

Ongoing …